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Two neutrino double beta decay within the ξ-approximation

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Abstract

We examine the contributions of odd-parity nuclear operators to the two-neutrino double beta decay $0^+ \rightarrow 0^+$ amplitude, which come from the P-wave Coulomb corrections to the electron wave functions and the recoil corrections to the nuclear currents. Although they are formally of higher order in $\alpha Z/2$ or v/c of the nucleon than the usual Fermi and Gamow-Teller matrix elements, explicit calculations performed within the QRPA show that they are significant when confronted with the experimental data.

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The two neutrino double beta decay ($2\nu\beta\beta$) $0^+ \rightarrow 0^+$ is the rarest process observed so far in nature. As such, it offers a unique opportunity for testing the nuclear physics techniques for half-lives $T^{2\nu}_{1/2} \sim (10^{19} - 10^{23})$ years [1, 2, 3, 4, 5, 6]. But, before resorting to a particular nuclear model, the expression for the $2\nu\beta\beta$ decay rate has to be figured out from the weak Hamiltonian. [1] Except for the work of Williams and Haxton [8], the allowed approximation (AA) has been universally adopted in all previous theoretical studies. This approximation is obtained in the long-wavelenth limit for the outgoing electrons and neutrinos by retaining only the Fermi (F) and Gamow-Teller (GT) operators in the non-relativistic impulse approximation for the hadronic current

$$J^\mu(\mathbf{r}) = \sum_n \tau^+_n \left[(g_V - g_A \mathbf{v}_n \cdot \boldsymbol{\sigma}_n) g^{\mu 0} + (g_A \sigma^k_n - g_V v^k_n) g^{\mu k}\right] \delta(\mathbf{r} - \mathbf{r}_n). \tag{1}$$

Here the operator τ^+ transforms a neutron into a proton and $\boldsymbol{\sigma}_n$, $\mathbf{r}_n$ and $\mathbf{v}_n$ stand, respectively, for the spin, the position and the velocity of the nucleon n. The well known result for the half-live is [9, 10, 11, 12]

$$\left[T^{2\nu}_{1/2}(0^+ \rightarrow 0^+)\right]^{-1} = \mathcal{G}_{2\nu}(W_0, Z) |\mathcal{M}_{2\nu}|^2, \tag{2}$$

where $\mathcal{G}(W_0, Z)$ is the phase space kinematical factor, and

$$\begin{aligned} \mathcal{M}_{2\nu} \equiv \mathcal{M}^A_{2\nu} &= -g^2_V \sum_N \frac{\langle 0^+_F | \tau^+ | 0^+_N \rangle \langle 0^+_N | \tau^+ | 0^+_I \rangle}{E_{0^+_N} - E_{0^+_I} + \frac{1}{2} W_0} \\ &+ g^2_A \sum_N \frac{\langle 0^+_F | \tau^+ \boldsymbol{\sigma} | 1^+_N \rangle \cdot \langle 1^+_N | \tau^+ \boldsymbol{\sigma} | 0^+_I \rangle}{E_{1^+_N} - E_{0^+_I} + \frac{1}{2} W_0}, \end{aligned} \tag{3}$$

is the nuclear matrix element in the AA, with all the notation having the usual meaning [10, 11].

The retardation of the $0^+ \rightarrow 0^+$ $2\nu\beta\beta$ transition rates mostly comes from the phase space factor. Yet, the nuclear correlations also slow down these processes. Their effect can be estimated by comparing the measured $|\mathcal{M}_{2\nu}|$ values, shown in table 1, with the naive

[1] The $2\nu\beta\beta$-decay is a second order nuclear weak transition, analogous to the electromagnetic nuclear double gamma decay. It could be interesting to notice that the half-life for the $\gamma\gamma$ $0^+_2 \rightarrow 0^+_1$ transition in ^{90}Zr is only $T^{2\gamma}_{1/2} \sim 1.4 \times 10^{-4}$ seconds [7].

estimate, $\mathcal{M}_{2\nu} \sim -0.3$ that is obtained within the closure approximation for an effective axial charge $g_A^{eff} = 1$ [9]. The resulting suppression factor ranges from 1/2 for ^{100}Mo to 1/15 for ^{130}Te.

The quasiparticle random phase approximation (QRPA) is the nuclear structure method most widely used to deal with the quenching of the moment $\mathcal{M}_{2\nu}$. We know that in this model the allowed moment $\mathcal{M}^A_{2\nu}$ is very sensitive to the ground state correlation (GSC) within the particle-particle (PP) channel. In a recent work [13] it has been shown both that: (a) the restoration of the isospin symmetry leads to $\mathcal{M}^A_{2\nu}(J^\pi = 0^+) \cong 0$, and (b) the exact QRPA calculations for $\mathcal{M}^A_{2\nu}(J^\pi = 1^+)$, when evaluated with a zero-range force, can be nicely fitted by the formula

$$\mathcal{M}^A_{2\nu}(J^\pi = 1^+) = \mathcal{M}^A_{2\nu}(J^\pi = 1^+; t = 0)\frac{1 - t/t_0}{1 - t/t_1}, \tag{4}$$

where $t = v_t^{pp}/v_s^{pair}$ is the ratio between the spin-triplet strength in the PP and the spin-singlet strength in the pairing channel. The free parameters t_0 and t_1 denote the zero and the pole parameters t_0 and t_1, with $t_1 \gtrsim t_0$, denote the zero and the pole of $\mathcal{M}^A_{2\nu}$, respectively. In the same work it has been suggested that this result is of general validity and that any modification of the nuclear hamiltonian or of the configuration space cannot lead to a different functional dependence. Several alterations of the QRPA have been proposed that might change that behavior. They include higher order RPA corrections [14], nuclear deformation [15], single-particle self-energy BCS terms [16] and particle number projection [17]. Yet, none of these amendments inhibits the matrix element $\mathcal{M}^A_{2\nu}$ to pass through zero near the "natural" value of the PP strength ($t \sim 1.5$). [2]

We also know that the GSC do not affect the $0\nu\beta\beta$-decay probability in the same way. The reason for that is that the exchange of the virtual neutrino in the $0\nu\beta\beta$ process gives rise to the neutrino potentials, which depend on the distance $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$ between the two vertices where the single β-decays take place. If one expands this dependence on r_{12} in

[2]When a physical quantity has a zero (or near zero) a conservative law should, very likely, be its origin. Thus, the behavior of $\mathcal{M}^A_{2\nu}$, described by (4), has been attributed to the partial restoration of the SU(4) Wigner symmetries [18]. This is not surprising since the operators $\tau^\pm\boldsymbol{\sigma}$ are infinitesimal generators of the $SU(4)$ group.

multipoles depending on $\mathbf{r}_1$ and $\mathbf{r}_2$, one sees that, besides exciting F and GT states, one is also going through virtual states with spin and parity $J^\pi \neq 0^+,\ 1^+$. The GSC are not so important for the forbidden transitions ($L > 0$) and thus they are not so drastically reducing the decay rate for the $0\nu\beta\beta$-decay as for the $2\nu\beta\beta$-decay. In fact, it turns out that for $t \sim 1.5$ the dominant contribution to $\mathcal{M}_{0\nu}$ comes from multipoles with $L > 0$ [13].

Motivated by the above arguments and by the hope that some favorable nuclear structure physics could enhance the higher order contributions to $\mathcal{M}_{2\nu}$, we study here the effects of the $2\nu\beta\beta$ matrix elements with $L = 1$. It will be assumed that the Coulomb energy of the electron at the nuclear radius is larger that its total energy. This leads to the ξ-approximation, which has been extensively utilized in the study of the first forbidden (FF) single β transitions [19]. Besides, it is well known that, when this approximation is applied, the FF observables can be expressed exactly as the allowed ones except for the replacements [9]

$$\tau^+ g_V \rightarrow -\tau^+ X; \qquad\qquad \tau^+ g_A \boldsymbol{\sigma} \rightarrow -\tau^+ \mathbf{Y}, \tag{5}$$

with

$$\begin{aligned} X &= g_A(\boldsymbol{\sigma}\cdot\mathbf{v} + \xi i\boldsymbol{\sigma}\cdot\mathbf{r}), \\ \mathbf{Y} &= g_V\mathbf{v} + \xi(g_V i\mathbf{r} - g_A\boldsymbol{\sigma}\times\mathbf{r}), \end{aligned} \tag{6}$$

where (in natural units)

$$\xi = \frac{\alpha Z}{2R} \cong 1.18\ ZA^{-1/3}, \tag{7}$$

Z and R being the charge and the radius of the daughter nucleus. [3] This result is obtained by retaining the velocity dependent terms in the hadronic current (1) and by considering only: i) the $S-$wave for the neutrino wave function $\phi^\nu(s)$, and ii) the $S-$ wave and the ξ correction term in the $P-$wave for the electron wave function $\psi^e(\mathbf{r}, s)$, i.e.,

$$\phi^\nu(s) = \frac{1}{\sqrt{2}}\begin{pmatrix} I \\ (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}) \end{pmatrix}\chi^\nu(s) \tag{8}$$

[3] We discuss here only the β^--decay. For positron emission the following changes are made: $Z \rightarrow -Z$ and $g_A \rightarrow -g_A$.

and

$$\psi^e(\mathbf{r}, s) = \sqrt{\frac{F_0(Z,\varepsilon)(\varepsilon+m)}{2\varepsilon}}\,[1 + i\xi(\boldsymbol{\alpha}\cdot\mathbf{r})]\begin{pmatrix} I \\ \frac{(\boldsymbol{\sigma}\cdot\mathbf{p})}{\varepsilon+m}\end{pmatrix}\chi^e(s), \tag{9}$$

where $F_0(Z,\varepsilon)$ is the Fermi factor and all the remaining notation is that of ref. [11].

Before proceeding let us recall that the contributions of all five FF moments are of the same order of magnitude. First, in the extreme single particle model

$$\frac{\langle f|\boldsymbol{\sigma}\times\mathbf{r}|i\rangle}{\langle f|i\mathbf{r}|i\rangle} = j_f(j_f+1) - l_f(l_f+1) - j_i(j_i+1) + l_i(l_i+1). \tag{10}$$

Next, the velocity dependent β-moments are related in a simple way to the corresponding position operators, when the harmonic oscillator radial wave functions are employed. One gets [19, 20, 21]:

$$\frac{\langle f|\boldsymbol{\sigma}\cdot\mathbf{v}|i\rangle}{\langle f|i\boldsymbol{\sigma}\cdot\mathbf{r}|i\rangle} = \frac{\langle f|\mathbf{v}|i\rangle}{\langle f|i\mathbf{r}|i\rangle} = (N_f - N_i)\omega_0, \tag{11}$$

where N and $\omega_0 \cong 80\ A^{-1/3}$ stand for the principal quantum number and the oscillator frequency, respectively. The most relevant single-particle transitions, in any nuclear model calculation, are those with $N_i = N_f + 1$ (see e.g., table 2 in ref. [20]). [4] Consequently the contributions of the operators $\boldsymbol{\sigma}\cdot\mathbf{v}$ and $\mathbf{v}$ always add destructively to those coming from the operators of $i\xi\boldsymbol{\sigma}\cdot\mathbf{r}$ and $i\xi\mathbf{r}$ (see eq. (6)). In addition, the ratio of the corresponding matrix elements is $\approx -68/Z$, i.e., of order of unity for medium heavy nuclei.

The conserved-vector-current (CVC) constraint leads to an alternative expression for the total matrix element $<\mathbf{v}>$ [22]

$$<\mathbf{v}> = -(2.4\xi + E_0) < i\mathbf{r}>, \tag{12}$$

where E_0 is the energy difference between the initial and final states. (In principle, this relation is exact as it includes exchange effects, induced interactions, etc. [19].) Besides, it is now well established [23] that the operator $\boldsymbol{\sigma}\cdot\mathbf{v}$ is enhanced by the one-pion exchange current contributions in nuclear medium [24]. Inspired by this effect, which can be accounted

[4] Note that the operators $i\boldsymbol{\sigma}\cdot\mathbf{r}$ ($i\mathbf{r}$) and $\boldsymbol{\sigma}\cdot\mathbf{v}$ ($\mathbf{v}$, $\boldsymbol{\sigma}\times\mathbf{r}$) have different Hermitian conjugation properties [expressed in equations (10) and (11)]. This makes that in a RPA calculation the signs of the ratios (10) and (11) are different for the forward and backward going contributions.

for by renormalizing the corresponding matrix element by the factor 1.7 $g_A^{free} \cong 2.0$ (see also ref. [21]), Williams and Haxton [8] have pointed out that the moment $< \boldsymbol{\sigma} \cdot \mathbf{v} >$ could have a significant effect on the rates for $2\nu\beta\beta$-decay. They also performed calculations in the Nilsson pairing model with the quadrupole-quadrupole core-polarization effect included. The result was to increase the AA predictions for $T_{1/2}^{2\nu}$ of ^{76}Ge, ^{82}Se, ^{128}Te and ^{130}Te nuclei by 15%, 25%, 15% and 20%, respectively (see also ref. [25]). Yet from the above discussion it is clear that the consequences of the remaining four FF moments have also be examined simultaneously. [5] Such an examination is the main topic of this paper.

The expression for the half-life for the $0^+ \rightarrow 0^+$ $2\nu\beta\beta$-decay, within the ξ-approximation, has been derived following the procedure of Doi et al. [11] and using the relations (8) and (9) for the lepton wave functions. A tedious but straightforward calculation [26] leads to the equation for the half-life that can again be cast in the form (1), with the *same kinematical factor* and the transition amplitude

$$\mathcal{M}_{2\nu} \equiv \mathcal{M}_{2\nu}^{\xi} = \mathcal{M}_{2\nu}^{A} + \mathcal{M}_{2\nu}^{FF}, \tag{13}$$

where

$$\begin{aligned} \mathcal{M}_{2\nu}^{FF} = & - \sum_N \frac{\langle 0_F^+ | \tau^+ X | 0_N^- \rangle \langle 0_N^- | \tau^+ X | 0_I^+ \rangle}{E_{0_N^-} - E_{0_I^+} + \frac{1}{2} W_0} \\ & + \sum_N \frac{\langle 0_F^+ | \tau^+ \mathbf{Y} | 1_N^- \rangle \cdot \langle 1_N^- | \tau^+ \mathbf{Y} | 0_I^+ \rangle}{E_{1_N^-} - E_{0_I^+} + \frac{1}{2} W_0}, \end{aligned} \tag{14}$$

is the contribution coming from the FF moments. It is also important to note that the shape of the electron spectrum is the same as in the AA.

As the spin singlet components predominate in the ground state of the even-even nuclei, the two terms in (3) and (14) have a tendency to sum up coherently. On the other hand, it is self-evident from relations (6) that, independently of the nuclear model employed in the evaluation of $\mathcal{M}_{2\nu}^{FF}$, the role played by the virtual states with $J^\pi = 0^-$ and $J^\pi = 1^-$ critically depend on the way in which the individual matrix elements combine to build up

[5] Williams and Haxton [8] have also hinted that, besides $< \boldsymbol{\sigma} \cdot \mathbf{v} >$, other parity-forbidden matrix elements should be considered. Particularly, because of the CVC relation, they have pointed towards the importance of $< \mathbf{v} >$.

the total matrix elements of X and $\mathbf{Y}$. Coherent combinations would strongly enhance the effect of $\mathcal{M}_{2\nu}^{FF}$. Yet, from the above discussion we have learned that the moments $< \boldsymbol{\sigma} \cdot \mathbf{v} >$ and $< i\boldsymbol{\sigma} \cdot \mathbf{r} >$, as well as $< \mathbf{v} >$ and $< i\mathbf{r} >$, always tend to add destructively. Hence the magnitude of $\mathcal{M}_{2\nu}^{FF}$ hangs crucially on the matrix element $< \boldsymbol{\sigma} \times \mathbf{r} >$ (both on its magnitude and its sign relative to the sign of $< \mathbf{v} >$). [6]

The numerical results for the matrix elements $\mathcal{M}_{2\nu}^{FF}$ are presented in table 2. They were obtained through the procedure adopted in our previous works [18], i.e., by using the same residual interaction, the same configuration space, etc. The moments labeled $\mathcal{M}_{2\nu}^{FF}(pair)$ were evaluated in the independent quasiparticle approximation, and those indicated by $\mathcal{M}_{2\nu}^{FF}(QRPA)$ within the QRPA. The matrix element $< \boldsymbol{\sigma} \cdot \mathbf{v} >$ has been estimated by using the relation (11), properly renormalized by the one-pion exchange currents. The CVC relation (12) has been used in the evaluation of the moment $< \mathbf{v} >$. The particle-hole interaction reduces the forbidden moments by roughly a factor of 2. But, at variance with the moments $\mathcal{M}_{2\nu}^{A}$, which are widely uncertain in the QRPA (due to the proximity of t_1 to t_0 in eq. (4)), the effect of the particle-particle interaction on $\mathcal{M}_{2\nu}^{A}$ is always very small. For all the nuclei, the contributions of the virtual states with $J^\pi = 0^-$, shown parenthetically in table 2, are smaller than those coming from the states with $J^\pi = 1^-$. There are two reasons for that. First, the individual single-particle matrix elements of the operator $\mathbf{Y}$ are usually larger than those of the operator X. This is caused mainly by the destructive interference between the individual FF matrix elements mentioned above. Among the former the most important are those with $\langle f|\boldsymbol{\sigma} \times \mathbf{r}|i\rangle = \langle f|i\mathbf{r}|i\rangle$, in which case $\langle f|\mathbf{Y}|i\rangle = \langle f|\mathbf{v}|i\rangle$ (for $g_A^{eff} = 1$). Second, for both type of transitions the partial contributions show a pronounced coherence, but the configuration space for the 1^- is bigger. For the sake of comparison, the matrix elements $\mathcal{M}_{2\nu}^{A}$ are displayed in table 2 also.

We note that the calculation done by Williams and Haxton [8] yields considerably larger

[6] It is worth noting that $\mathcal{M}_{2\nu}^{FF}$ can be derived from $\mathcal{M}_{2\nu}^{A}$ by performing the substitution (5) in (3), i.e., in the analogous way that are obtained the FF observables from the allowed ones in the simple β decay. This common future of the single and double beta decays came somewhat as a surprise. One should remember here that, because of the antisymmetrization in the momentum and spin-orientation, the $2\nu\beta\beta$ amplitude is not just a simple product of two single β decays. Yet, a careful examination of the analytic calculations shows that it is precisely the anti-symmetrization procedure that establishes such a correspondence.

contributions of the $J^\pi = 0^-$ states that our QRPA evaluation. For instance, for the $^{82}Se \rightarrow\ ^{82}Kr$ and $^{130}Te \rightarrow\ ^{130}Xe$ transitions they get $\mathcal{M}_{2\nu}^{FF}(0^-) \cong 0.010$. The discrepancy with our results can be attributed only partially to the effect of the matrix element $< i\boldsymbol{\sigma}\cdot\mathbf{r} >$, which has not been considered in ref. [8]. By neglecting the last one we obtain that $\mathcal{M}_{2\nu}^{FF}(0^-) \cong 0.006$ for both nuclei. The remaining discrepancy has to be attached to the difference in the nuclear models employed in ref. [8] and here.

From the results shown in table 2 it is clear that in the QRPA the matrix elements $\mathcal{M}_{2\nu}^{FF}$ are significant when compared with the $\mathcal{M}_{2\nu}^{A}$ moments. But, the real relevance of $\mathcal{M}_{2\nu}^{FF}$ comes from the confrontation with the experimental data. In fact, we see that they are roughly comparable to $|\mathcal{M}_{2\nu}|(exp)$ needed to explain the half lives of ^{128}Te, ^{130}Te, and ^{150}Nd nuclei. For the remaining four nuclei displayed in tables 1 and 2 the moments $\mathcal{M}_{2\nu}^{FF}$ are relatively small in comparison with the measured amplitudes. Yet, even in this case the parity forbidden moments might become significant in the theoretical evaluation of the $2\nu\beta\beta$-decay half-lives, as they have a tendency to cancel against the allowed moments. Besides, the sensitivity of $2\nu\beta\beta$ experiments has been greatly improved in recent years, and there is no reason to believe that further improvements are not still possible.

In closing, we point out that there are other forbidden terms that should be studied. For instance, we are in process of examining the contributions arising from virtual states $J^\pi = 2^-$, which might modify both the electron spectrum shape and the $\mathcal{G}(W_0, Z)$ factor predicted by the AA. This could be of relevance for the excess of high-energy electrons seen in the $\beta\beta$ spectra and presently ascribed to the majoron emission. Finally, we notice that in the new class of majoron models the $\beta\beta$-decay proceeds via the odd-parity nuclear operators [27].

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Tables

Table 1: The measured half-lives $T_{1/2}^{2\nu}$ and the phase space function $G_{2\nu}(W_0, Z)$, which were used to extract the values of $|\mathcal{M}_{2\nu}|(exp)$, given in natural units.

Nucleus	$T_{1/2}^{2\nu}[yr\ 10^{20}]$	$G^{2\nu}[yr]^{-1}$	$\lvert\mathcal{M}_{2\nu}\rvert(exp)$
^{76}Ge	$14.3 \pm 0.17\,^{a)}$	$5.39\ 10^{-20}$	0.114 ± 0.007
^{82}Se	$1.08^{+0.26}_{-0.06}\,^{b)}$	$1.80\ 10^{-18}$	$0.071^{+0.002}_{-0.007}$
^{96}Zr	$0.39 \pm 0.09\,^{c)}$	$7.66\ 10^{-18}$	$0.058^{+0.008}_{-0.006}$
^{100}Mo	$0.115^{+0.030}_{-0.020}\,^{d)}$	$3.91\ 10^{-18}$	$0.149^{+0.015}_{-0.017}$
^{128}Te	$(7.7 \pm 0.4)\ 10^{4\,e)}$	$3.53\ 10^{-22}$	0.019 ± 0.001
^{130}Te	$27 \pm 1\,^{e)}$	$1.98\ 10^{-18}$	0.014 ± 0.001
^{150}Nd	$0.17^{+0.14}_{-0.09}\,^{f)}$	$4.91\ 10^{-17}$	$0.035^{+0.014}_{-0.009}$

^{a}) (laboratory data) ref. [1]
^{b}) (laboratory data) ref. [2]
^{c}) (geochemical data) ref. [3]
^{d}) (laboratory data) ref. [4]
^{e}) (geochemical data) ref. [5]
^{f}) (laboratory data) ref. [6]

Table 2: Calculated allowed and first forbidden matrix elements (in natural units) for an effective axial charge $g_A^{eff} = 1$. The column labeled $\mathcal{M}_{2\nu}^{FF}(pair)$ is evaluated using the independent quasiparticle approximation, while the column $\mathcal{M}_{2\nu}^{FF}(QRPA)$ includes the effects of both the particle-hole and the particle-particle interactions. The values in parenthesis stand for the contributions of virtual states with $J^\pi = 0^-$ to the total moments.

Nucleus	$\mathcal{M}_{2\nu}^{FF}(pair)$		$\mathcal{M}_{2\nu}^{FF}(QRPA)$		$\mathcal{M}_{2\nu}^{A}(QRPA)$
^{76}Ge	-0.015	(-0.005)	-0.008	(-0.004)	0.050
^{82}Se	-0.019	(-0.006)	-0.009	(-0.004)	0.060
^{96}Zr	-0.033	(-0.010)	-0.014	(-0.006)	0.010
^{100}Mo	-0.035	(-0.010)	-0.014	(-0.006)	0.051
^{128}Te	-0.020	(-0.004)	-0.012	(-0.003)	0.059
^{130}Te	-0.019	(-0.004)	-0.012	(-0.002)	0.048
^{150}Nd	-0.060	(-0.009)	-0.031	(-0.008)	0.033